

Pensieve header: A Brave new \$V\$.

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
  Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
  sd_SeriesData => MapAt[Expand, sd, 3]
];
βCollect[B[ω_, μ_]] := B[βSimplify[ω], βSimplify[μ]];
{V0, C0, sol} = Get[Switch[$PerturbativeDegree,
  4, "SolutionToDegree4-120523.m",
  6, "SolutionToDegree6-120523.m",
  8, "SolutionToDegree8-120524.m"
]];
C = C0 /. κ1 → 0;
v = B[Series[ $\frac{\text{Sinh}[c_1 \hbar / 2]}{c_1 \hbar / 2}$ , {ħ, 0, $PerturbativeDegree}], 0];
Φ0 = (Inverse[V0] // dP[12, 3]) ** Inverse[V0] ** (V0 // dP[2, 3]) ** (V0 // dP[1, 23]);
V = (C // dP[12]) ** V0 ** Inverse[C ** (C // dP[2])];
Φ = (Inverse[V] // dP[12, 3]) ** Inverse[V] ** (V // dP[2, 3]) ** (V // dP[1, 23]);
CC = C ** C;
Clear[C];
```

In[15]= Φ == Φ0

Out[15]= True

```

In[16]:= {
  "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
  "TwistEq" → V ** θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
  "Unitarity" →
    V ** (Inverse[CC // dP[12]]) ** (V // dA[1] // dA[2]) == Inverse[CC ** (CC // dP[2])],
  "VerticalFlipForV" → V ** (Inverse[CC // dP[12]]) ** (V // dS[1] // dS[2]) ==
    R[1, 2] ** Inverse[CC ** (CC // dP[2])],
  "CapEquation" → (V // dcap[1] // dcap[2]) == B[1, 0],
  "VSidesDelete" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
  "CapsAndCups" → CC == (CC // dS[1]),
  "Pentagon" → Ⓢ ** (Ⓢ // dP[1, 23, 4]) ** (Ⓢ // dP[2, 3, 4]) ==
    (Ⓢ // dP[12, 3, 4]) ** (Ⓢ // dP[1, 2, 34]),
  "PositiveHexagon" → (θ[1, 2, +1] // dP[12, 3]) ==
    (Ⓢ ** θ[2, 3, +1] ** Inverse[Ⓢ // dP[1, 3, 2]] ** θ[1, 3, +1] ** (Ⓢ // dP[3, 1, 2])),
  "NegativeHexagon" → (θ[1, 2, -1] // dP[12, 3]) ==
    (Ⓢ ** θ[2, 3, -1] ** Inverse[Ⓢ // dP[1, 3, 2]] ** θ[1, 3, -1] ** (Ⓢ // dP[3, 1, 2])),
  "HorizontalFlipForⓈ" → Ⓢ ** (Ⓢ // dP[3, 2, 1]) == B[1, 0],
  "VerticalFlipForⓈ" → Ⓢ ** (Ⓢ // dS[1] // dS[2] // dS[3]) == B[1, 0],
  "OverhandEquation" →
    (Ⓢ // dΔ[1, 0, 1] // dS[2] // dS[3] // dm[0, 3, 0] // dm[1, 2, 1]) == B[1, 0],
  "ValueOfv" → (Ⓢ // dS[2] // dm[3, 2, 2] // dm[2, 1, 1]) == v,
  "ValueOfCC" → Inverse[CC ** CC] == v,
  "VTopDelete" →
    (V // dS[2] // dm[1, 2, 1]) == (R[1, 1, -1/2] // dS[1]) ** Inverse[CC ** CC],
  "EKTopCapLeftPuncture" →
    (V // tη[1] // dm[2, 3, 2] // dS[2] // hm[1, 2, 1]) == B[1, 0],
  "EKRightCupLeftPuncture" →
    (V // dm[3, 2, 2] // hη[2] // tη[1] // dm[1, 2, 1]) == B[1, 0],
  "EKRightCupTopPuncture" → (V // dm[3, 2, 2] // hη[2] // dS[1] // dm[2, 1, 1]) ==
    Inverse[CC ** CC],
  "EKTopCapRightPuncture" → (V // tη[2] // dm[1, 3, 1] // dS[1] // dm[2, 1, 1]) ==
    R[1, 1, -1/2],
  "EKLeftCupRightPuncture" → (V // dm[3, 1, 1] // hη[1] // tη[2] // dm[2, 1, 1]) ==
    R[1, 1, 1/2],
  "EKLeftCupTopPuncture" → (V // dm[3, 1, 1] // hη[1] // dS[2] // dm[1, 2, 1]) ==
    (R[1, 1, -1/2] // dS[1]) ** Inverse[CC ** CC]
} //
ColumnForm

```

```
Out[16]= R4 → True
TwistEq → True
Unitarity → True
VerticalFlipForV → True
CapEquation → True
VSidesDelete → True
CapsAndCups → True
Pentagon → True
PositiveHexagon → True
NegativeHexagon → True
HorizontalFlipForΦ → True
VerticalFlipForΦ → True
OverhandEquation → True
ValueOfv → True
ValueOfCC → True
VTopDelete → True
EKTopCapLeftPuncture → True
EKRightCupLeftPuncture → True
EKRightCupTopPuncture → True
EKTopCapRightPuncture → True
EKLeftCupRightPuncture → True
EKLeftCupTopPuncture → True
```

```
In[17]:= buckle = (Inverse[Φ] // dP[13, 2, 4]) **
(Φ // dP[1, 3, 2]) ** θ[3, 2] ** Inverse[Φ] ** (Φ // dP[12, 3, 4])
```

A very large output was generated. Here is a sample of it:

( <<1>> )

Show Less Show More Show Full Output Set Size Limit...

```
In[18]:= LuckyV = buckle // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]
```

$$\text{Out[18]= } \begin{pmatrix} 1 & h[1] \\ t[1] & \frac{c_2 \hbar}{24} + \left(-\frac{7 c_1^2 c_2}{5760} - \frac{7 c_1 c_2^2}{5760} - \frac{c_2^3}{1440}\right) \hbar^3 + \left(\frac{31 c_1^4 c_2}{967680} + \frac{31 c_1^3 c_2^2}{483840} + \frac{83 c_1^2 c_2^3}{967680} + \frac{13 c_1 c_2^4}{241920} + \frac{c_2^5}{60480}\right) \hbar^5 + O[\hbar]^7 & \frac{1}{2} + \left(\frac{c_1}{8} + \right. \\ t[2] & \left. - \frac{c_1 \hbar}{24} + \left(\frac{7 c_1^3}{5760} + \frac{7 c_1^2 c_2}{5760} + \frac{c_1 c_2^2}{1440}\right) \hbar^3 + \left(-\frac{31 c_1^5}{967680} - \frac{31 c_1^4 c_2}{483840} - \frac{83 c_1^3 c_2^2}{967680} - \frac{13 c_1^2 c_2^3}{241920} - \frac{c_1 c_2^4}{60480}\right) \hbar^5 + O[\hbar]^7 \right. \end{pmatrix}$$

```
In[19]:= V
```

$$\text{Out[19]= } \begin{pmatrix} 1 - \frac{1}{24} (C_1 C_2) \hbar^2 + \left(\frac{c_1^2 c_2}{1440} + \frac{11 c_1^2 c_2^2}{5760} + \frac{c_1 c_2^3}{1440}\right) \hbar^4 + \left(-\frac{c_1^5 c_2}{60480} - \frac{17 c_1^4 c_2^2}{241920} - \frac{107 c_1^3 c_2^3}{967680} - \frac{17 c_1^2 c_2^4}{241920} - \frac{c_1 c_2^5}{60480}\right) \hbar^6 + O[\hbar]^7 \\ t[1] & \frac{c_2 \hbar}{24} \\ t[2] & -\frac{c_1}{2} \end{pmatrix}$$

```
In[20]:= V == LuckyV ** Inverse[CC (CC // dP[2])] ** (CC // dP[12])
```

```
Out[20]= True
```